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DETECTION OF CRACKS IN CANTILEVER BEAMS: EXPERIMENTAL SET-UP USING OPTICAL TECHNIQUES AND THEORETICAL MODELLING

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1. INTRODUCTION

When a structure experiences a certain degree of damage its stiffness and inertial properties suffer a change, and a new state of force distribution and equilibrium is attained. The variations of these mechanical parameters produce alterations of the natural frequencies and mode shapes of the virgin structural system. The ultimate goal of vibration monitoring techniques is the knowledge of the structural health of the element under study by measuring its dynamical response. In a large majority of cases, damage is caused by the appearance of cracks that originate discontinuities in the structure.

From a historical viewpoint, the effect of cracks on reinforced concrete structural elements investigated by Bock [1] is probably the first situation where the effect of cracks upon the dynamic properties of a structural element was analyzed. A detailed survey of the literature can be found in references [2–4].

Structural health assessment through dynamical analysis has been proved to be successful in several cases, as it is the case of cracked rotors. Recently, Ramesh *et al.* [5] analyzed the dynamic behavior of annular plates with periodic radial cracks, in an ingenious attempt to model cracked flywheels, clutch plates, etc.

In this paper, we will analyze the dynamic behavior of cracked cantilever beams. The effect of cracks on the dynamic behavior of beams has received much attention because of its importance in mechanical and civil engineering applications [6]. It is interesting to point out that in the case of a long, fractured bone the variation of natural frequencies may eventually constitute an indication of the healing process [7].

Several theoretical methods have been developed to model the problem of cracked beams. Theoretical approaches found in the literature range from analytical to numerical. Gudmunson [8] used a first order perturbation method

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to predict the changes in resonance frequencies of a structure resulting from cracks, notches or other geometrical changes. Qian *et al.* [9] used the finite element method to model the stress in a cracked beam. In both papers, theoretical predictions are compared with the experimental measurements reported by Wendtland [10].

Ostachowicz and Krawczuk [11] investigated the influence of the position and depth of two open cracks upon the fundamental frequency of the natural flexural vibrations of a cantilever beam. To model the effect of the local stress in the crack, they introduced two different functions according to the symmetry of the crack.

Narkis [12] simulated the crack by an equivalent spring, connecting the two segments of the beam. Based on this model, he developed a closed-form solution which he applied to the study of the inverse problem of localization of cracks on the basis of frequency measurements. Using a similar model Masoud *et al.* [13] investigated the transverse vibrational characteristics of a prestressed fixed–fixed beam with a symmetric crack in its middle span and the coupling effect between the crack depth and the axial load on the fundamental frequency. They showed that it is not possible to treat this problem by superposition of the two separate effects (crack plus axial load). They also reported experimental results obtained using piezoelectric accelerometers to measure the vibration signal for a fixed–fixed beam.

Ruotolo and Surace [14] developed a method that uses the modal parameters of the lower modes for the non-destructive detection and sizing of cracks in beams.

A variational approach to the problem of cracked beams has been used by Chondros *et al.* [15], who developed a continuous cracked beam vibration theory for the lateral vibration of cracked Bernoulli–Euler beams with single-edge or double-edge open cracks. In that paper, the vibrational formulation is used to develop the differential equation and the boundary conditions of the cracked beam as a one-dimensional continuum. They also reported experimental results for the variation of the fundamental frequency of a simply supported beam with a fatigue crack in its middle span.

Several authors have also studied the non-linear characteristics of open–closed cracks. Chati *et al.* [16] studied the non-linear case of open–closed cracks. They solved it by the Finite Element Method, introducing a bilinear frequency. Tsyfansky and Beresnevich [17] analyzed the essential non-linear properties of open–closed cracks as a way to detect the presence of the crack, as this non-linear behavior is not present in the undamaged structure.

In the work presented herein, we report measurements made using an optical experimental set-up which includes a He–Ne laser and a photodiode. The principles used in the measuring process are explained. A simple, one-dimensional theoretical model is used to simulate the dynamical behavior of the beam. Two different local flexibility non-dimensional functions are tried out and the theoretical predictions are compared with the experimental results. Theoretical predictions and experimental results show that very good agreement is achieved for cracks as deep as 80% of the total height of the beam.

2. EXPERIMENTAL SET-UP

In the experiments described herein, we measured the first five natural frequencies of the cantilever beam in Figure 1, where the cracks were artificially produced by an end mill. The same methodology may be employed to detect frequency variations in beams with fatigue cracks. The laboratory set-up used for the measurements is shown in Figure 2. The purpose of this experimental arrangement is to determine the natural frequencies of cracked cantilever beams from measurements of the oscillation amplitude at a fixed position along the beam. As the frequency of the external excitation is swept across the range of interest, the eigenfrequencies are identified as those frequency values for which a local maximum in the oscillation amplitude is found, i.e., where resonant coupling occurs. In our set-up, the beam of a 15-m W He–Ne laser emitting in the TEM_{00} mode (Type: Melles Griot 05-LHR-991) is intercepted by a knife edge attached to the cantilever beam and focused by a small lens into a photodiode. Interception of

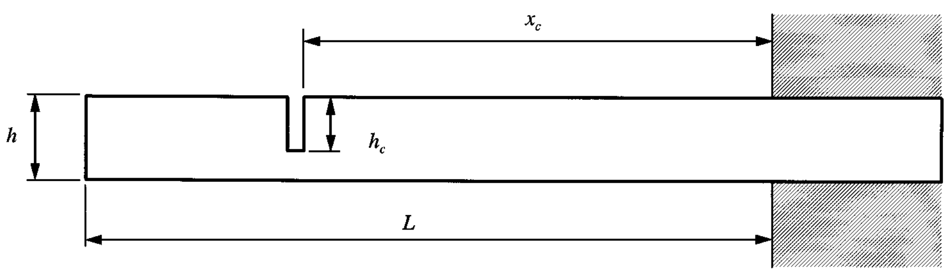

Figure 1. Cantilever beam under study.

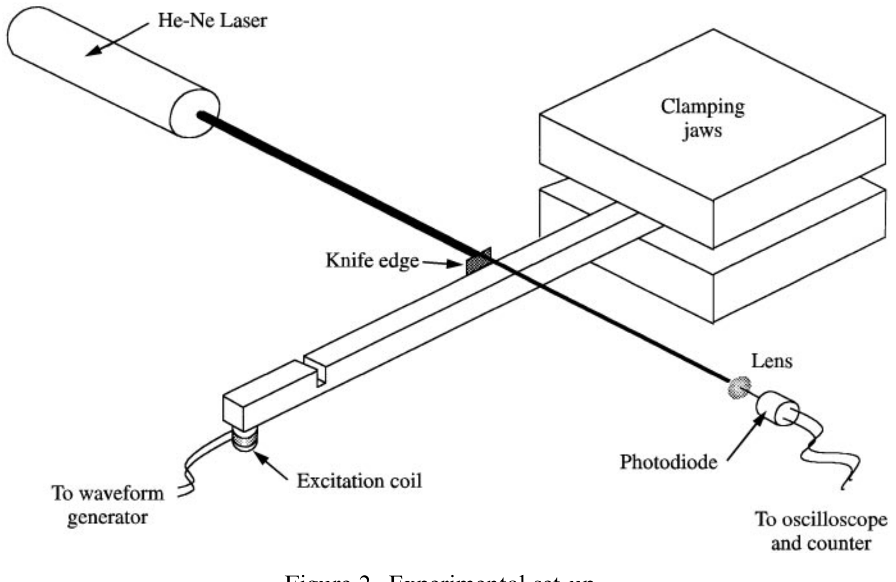

Figure 2. Experimental set-up.

the laser beam by the blade results in a decrease in the light intensity collected by the photodiode. As the TEM_{00} mode in which the He–Ne laser emits exhibits a Gaussian irradiance profile [18], the output signal of the photodiode varies as the error function when the knife edge is moved. Since the error function is approximately linear in the neighborhood of the origin, the linear range of the optical measuring system is then found close to the position where the blade intercepts half of the laser beam. The output signal from the photodiode was recorded by a 60 MHz digital real-time oscilloscope (Type: Tektronix TDS 210) and simultaneously sent to a 75 MHz timer/counter (Type: Hewlett-Packard 5308A). This simple, yet precise set-up allowed for the measurement of local displacements as small as 1 μm.

The cantilever beams were forced into oscillation by a 15-mm-diameter, 20-turn coil weighting 2·8 gr. attached to the end of the beam. The coil was placed in the field of a permanent magnet and was excited by a waveform generator (Type: Hewlett-Packard 209A).

The cantilever beams used in our experiments had square cross-sections and were cut from bars of commercial aluminum alloy (Type: 2420). In all cases, the length of the beams was measured to be $L = 400{\cdot}3 \pm 0{\cdot}3$ mm. Beam height varied from $9{\cdot}30 \pm 0{\cdot}05$ to $9{\cdot}45 \pm 0{\cdot}05$ mm among beams, depending on the aluminum stock. Cracks were cut using a 2·5 mm end mill, which resulted in crack-width-to-beam-length ratios of $(6{\cdot}5 \pm 0{\cdot}5) \times 10^{-3}$. Clamping of the beams was secured through the use of two heavy steel jaws. The whole experimental set-up allowed for reproducibility of the frequency measurements well within 0·5%.

3. MATHEMATICAL MODELLING

In the physical system under consideration, shown in Figure 1, the beam has a uniform rectangular cross-section and the crack is located at position x_c. In the case of harmonic flexural vibrations of Bernoulli–Euler beams, the following non-dimensional equation arises:

$$\frac{\mathrm{d}^4 W(X)}{\mathrm{d}X^4} - \Omega^2 W(X) = 0 \tag{1}$$

In the latter expression, the non-dimensional variables are defined as

$$X = \frac{x}{L}, \quad \Omega^2 = \omega^2 \frac{A_0 L^4 \rho}{EI},$$

where L is the total length of the beam, A_0 is the cross-sectional area, ρ is the mass density, E is the modulus of elasticity of the beam, and I is the area moment of inertia for the beam cross-section. Due to the localized crack effect, the cracked beam can be simulated as two uniform beams joined together by a torsional spring at the crack location [12]. The problem has then an exact solution given by

$$W(X) = \begin{cases} A_1 \cos(\gamma X) + B_1 \sin(\gamma X) + \\ C_1 \cosh(\gamma X) + D_1 \sinh(\gamma X) \quad \text{for } X < x_c/L, \\ \\ A_2 \cos(\gamma X) + B_2 \sin(\gamma X) + \\ C_2 \cosh(\gamma X) + D_2 \sinh(\gamma X) \quad \text{for } X > x_c/L \end{cases} \tag{2}$$

with $\Omega = \gamma^2$ and $X = x/L$. In the case under study, the boundary conditions in both ends of the beam are

$$W_1(0) = W_1'(0) = 0, \quad W_2''(1) = W_2'''(1) = 0 \tag{3}$$

The non-dimensional local flexibility can be computed from the strain energy function, and is given as a function of both the ratio between the crack depth and the beam thickness, $H = h_c/h$, and the height to length ratio, $e = h/L$, with h the height of the beam. The continuity between the modes to the left and to the right of the crack implies the following conditions at the crack location:

$$W_1(X_c) = W_2(X_c), \qquad W_2'(X_c) - W_1'(X_c) = \Theta W_1''(X_c),$$
$$W_1''(X_c) = W_2''(X_c), \qquad W_2'''(X_c) = W_1'''(X_c), \tag{4}$$

where $\Theta(e, H)$ is the non-dimensional flexibility function. Different functional forms have been proposed, according to the symmetry of the crack [11, 13, 15]. In this paper, we used the functional form proposed by Ostachowicz and Krawczuk [11] for the case of non-symmetrical cracks, and the one proposed by Chondros *et al.* [15] in their continuous vibration theory approach. These functions are, respectively,

$$\Theta_1(e, H) = 3\pi e H^2 \times (0{\cdot}6384 - 1{\cdot}035\, H + 3{\cdot}5\, H^2 - 5{\cdot}1773\, H^3 + 7{\cdot}553\, H^4 - 7{\cdot}332\, H^5 + 2{\cdot}4909\, H^6) \tag{5}$$

and

$$\Theta_2(e, H) = 3\pi e H^2 \times (0{\cdot}6272 - 1{\cdot}04533\, H + 4{\cdot}5948\, H^2 - 9{\cdot}9736\, H^3 + 20{\cdot}2948\, H^4 - 33{\cdot}0351\, H^5 + 47{\cdot}1063\, H^6 - 40{\cdot}7556\, H^7 + 19{\cdot}6\, H^8) \cdot \tag{6}$$

It is important to note that the flexibility due to the crack is not distributed over the length of the beam as supposed in reference [14], but it is localized in the crack position. In spite of this, an excellent agreement with the experimental values is obtained.

4. RESULTS AND CONCLUSIONS

Figures 3 and 4 and Table 1 show results of both experimental determinations and theoretical modelling. Figure 3 depicts the crack's second modal frequency for

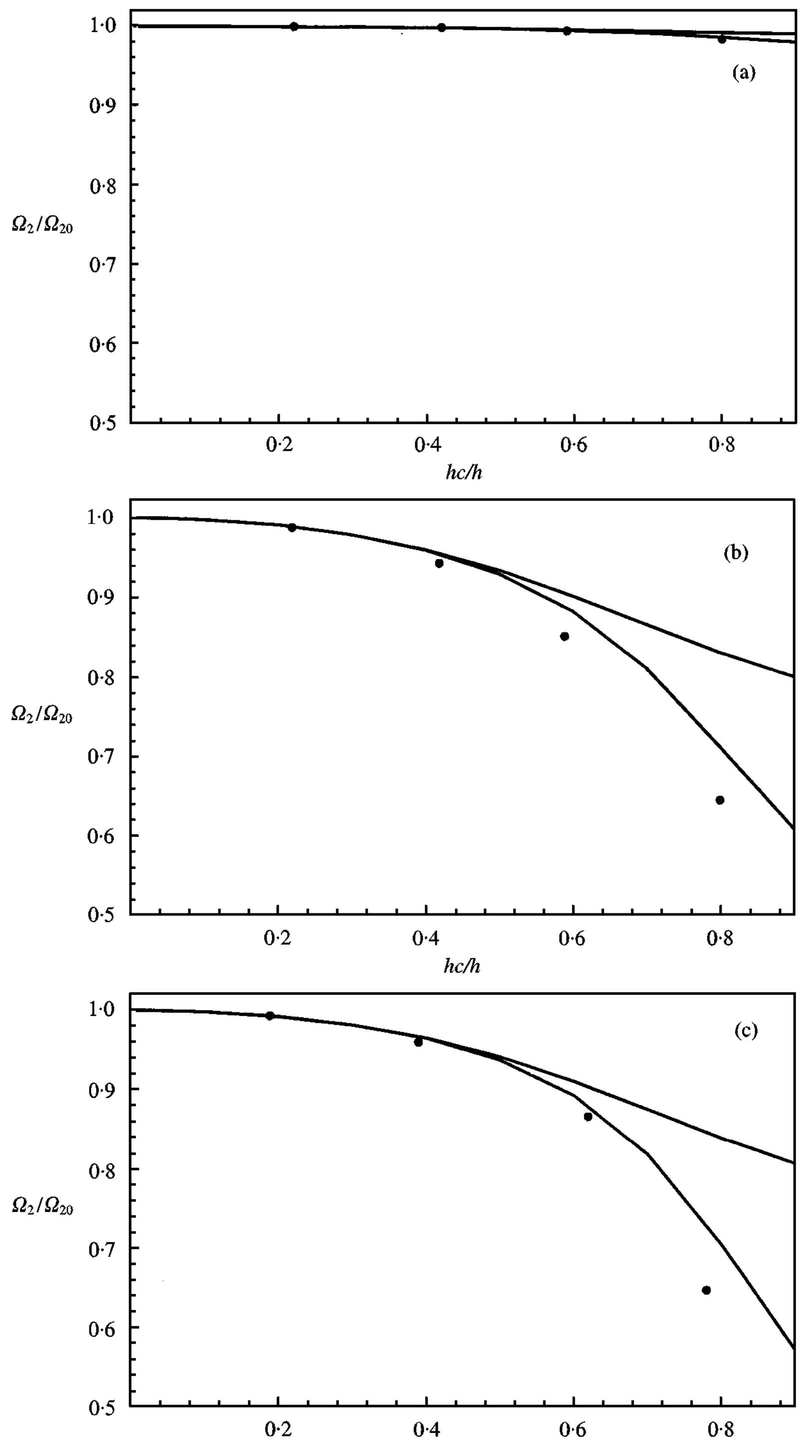

Figure 3. Comparison between the second resonance frequency obtained experimentally and those calculated, for different positions and depths of the crack using both Θ functions. Dots are experimental results. The error in the determination of the frequency ratio is of the order of 1%. In all figures, upper curves correspond to function Θ_1 and lower curves correspond to function Θ_2. (a) $X_c = 0{\cdot}25$, (b) 0·5, (c) 0·63.

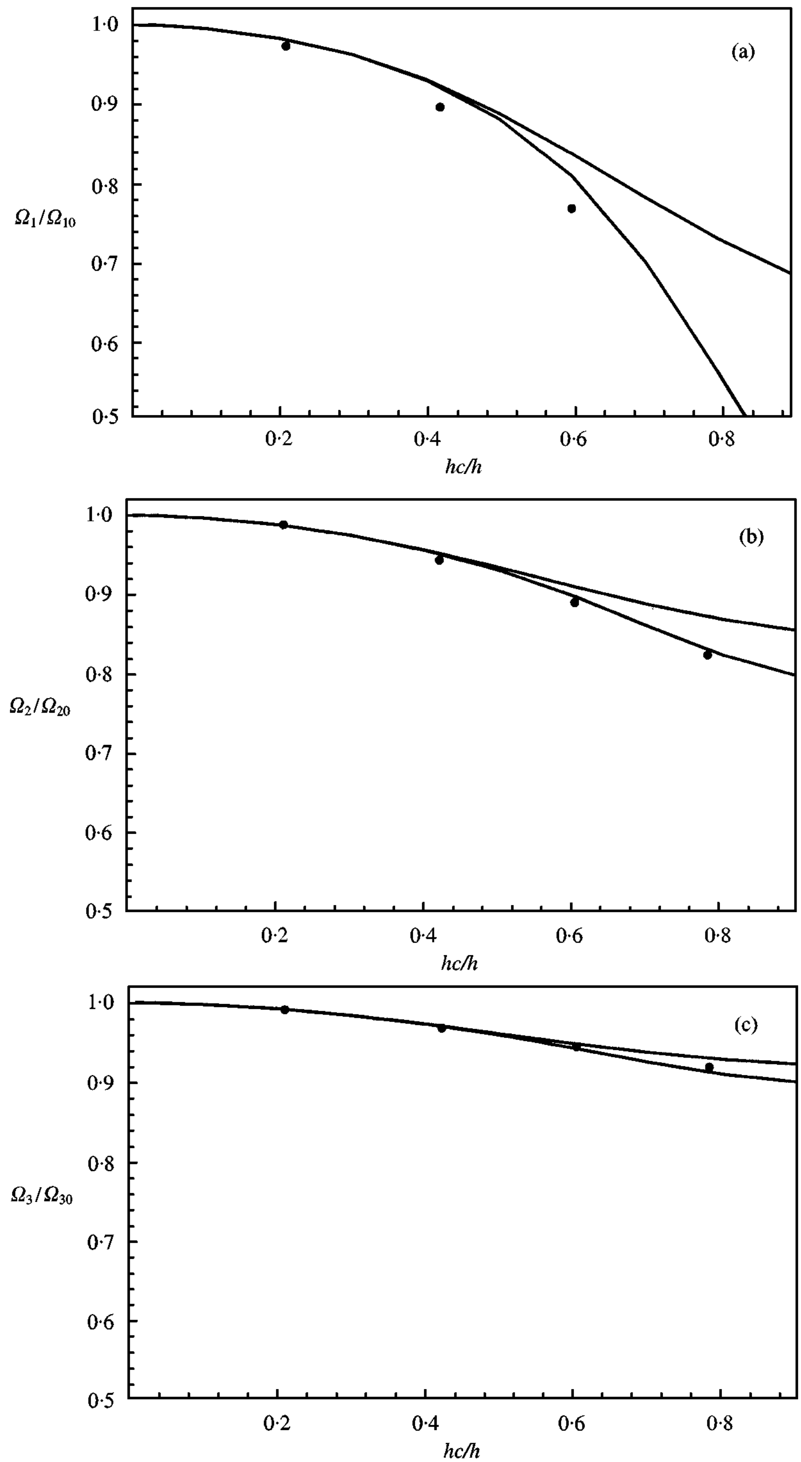

Figure 4. Comparison between the experimental values and the mathematical model for different modal frequencies. The crack is located at $x = 0{\cdot}049$. (a) Ω_1/Ω_{10}, (b) Ω_2/Ω_{20}, (c) Ω_3/Ω_{30}.

TABLE 1

Normalized non-dimensional frequencies of a cantilever beam

xc/L	hc/h	$f1$	$f2$	$f3$	$f4$	$f5$
0·05	0·21	0·97	0·99	0·99	1·00	1·00
	0·42	0·90	0·94	0·97	0·99	0·99
	0·60	0·77	0·89	0·95	0·98	0·99
	0·78	0·48	0·82	0·92	0·96	0·96
0·13	0·19	0·98	1·00	1·00	1·00	0·99
	0·39	0·94	0·99	1·00	0·99	0·97
	0·62	0·80	0·98	1·00	0·97	0·90
	0·78	0·54	0·96	1·00	0·94	0·80
0·25	0·22	0·99	1·00	0·99	0·99	1·00
	0·42	0·95	1·00	0·95	0·96	0·99
	0·59	0·85	0·99	0·90	0·92	0·98
	0·80	0·59	0·98	0·79	0·86	0·98
0·50	0·22	1·00	0·99	1·00	0·99	1·00
	0·42	0·98	0·94	1·00	0·96	1·00
	0·59	0·95	0·85	0·99	0·89	1·00
	0·80	0·81	0·64	0·99	0·77	0·99
0·63	0·19	1·00	0·99	0·99	1·00	0·99
	0·39	1·00	0·96	0·97	1·00	0·96
	0·62	1·00	0·87	0·91	1·00	0·91
	0·78	0·94	0·65	0·83	0·99	0·82
0·71	0·21	0·99	0·99	0·98	1·00	0·99
	0·42	0·99	0·96	0·93	0·98	0·99
	0·60	0·98	0·89	0·85	0·96	0·98
	0·78	0·96	0·68	0·72	0·94	0·95

different locations and depths of the crack. Figure 4 shows the changes on the modal frequencies as a function crack's depth for a crack location close to the clamping. Table 1 summarizes the experimental results for the determination of the lower five non-dimensional normalized natural frequencies of the cantilever beam under study. The normalization factor is the corresponding modal frequency of the virgin beam.

Data shows that the influence of a crack with a given depth on the natural frequency of the beam strongly depends on the crack's location. When the crack is at a position where the modal shape of the virgin beam has a small curvature, it produces a very small effect on the natural frequency. Conversely, at those locations where the curvature of the modal shape is large, the effect of the crack is easily detected. For example, for the second modal shape there is only one node placed at $x/L = 0{\cdot}78$. The curvature is null at $x/L = 0{\cdot}22$ and maximum at $x/L = 0{\cdot}52$. Accordingly, Table 1 reveals that the change in the second modal frequency is maximum when the crack is located at $x/L = 0{\cdot}5$ (near the point of maximal curvature) and minimum when the crack is located at $x/L = 0{\cdot}25$ (near the point of zero curvature).

A comparison between the experimental values and the theoretical results shows a good agreement for crack depths of up to 80% of the total height on both flexibility functions. However, it is evident from the graphs that the best approach for $H > 0{\cdot}6$ is the one proposed in reference [15].

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